

Unit 4 / Day 4

Sketching Graphs of Polynomial Functions

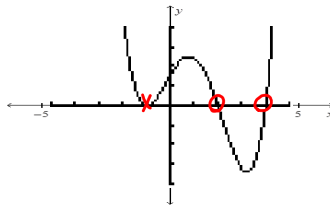
Objective:

Students will describe the graph and state the x-intercepts of a polynomial given its equation in factored form.

Unit 4 / Day 4

1. Graph the following equation in your graphing calculator: $y = x^4 - 4x^3 - 3x^2 + 10x + 8$

Change the WINDOW to Xmin = -5 Xmax = 5 Xscl = 1
Ymin = -20 Ymax = 20 Yscl = 5



- a. What relationship do you notice between the graph and the factored form of the equation?

The **factored form** of this polynomial is:

$$y = (x+1)(x+1)(x-2)(x-4) = (x+1)^2(x-2)(x-4)$$

$x = -1$ bounce

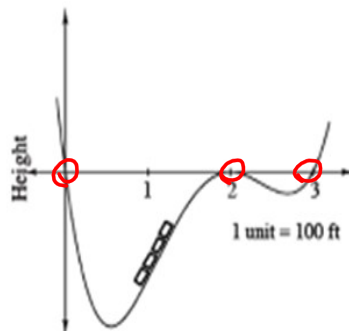
$x = 2$ crosses

$x = 4$ crosses

- b. Look specifically at $x = -1$ on the graph. Why do you think the graph does this?

bounces $(x+1)^2$

2. The Mathematical Carnival Company has decided to build a new roller coaster with a special feature: part of the ride will go underground. The designers will use polynomial functions to describe different pieces of the track. Part of the ride is shown. Write a possible equation for this ride.



$x=0$ cross

$x=2$ bounce

$x=3$ cross

$$y = x(x-2)^2(x-3)$$

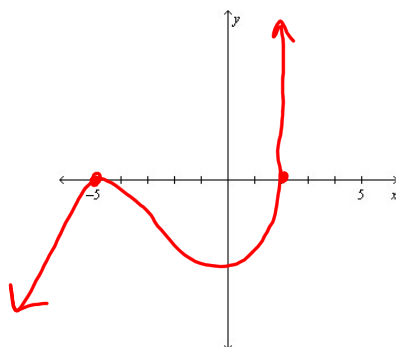
3. Examine the polynomial: $y = (x - 2)(x + 5)^2$

- a. What can you determine about the graph based on its equation?

$x=2$ cross

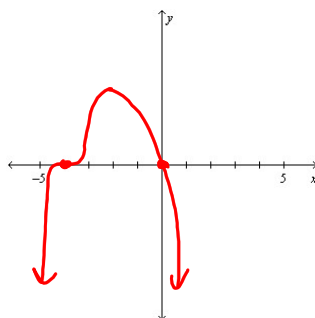
$x=-5$ bounce

- b. Sketch the graph (use your calculator to help).



4. Examine the polynomial: $y = -x(x+4)^3$

a. Graph the function.



b. How does the odd exponent change the graph?

flatten

c. How does the negative change the graph?

right arm goes down

CONCLUSIONS:

For an equation in factored form...

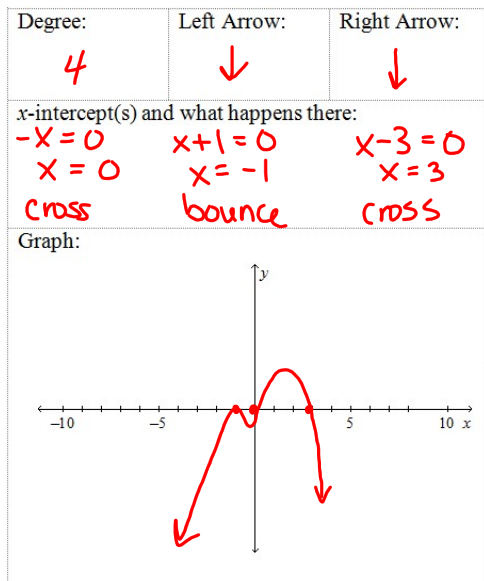
... when the factor is $(x \pm \#)^1$,
the graph crosses the x -axis.

... when the factor is $(x \pm \#)^{\text{even exponent}}$,
the graph bounces the x -axis.

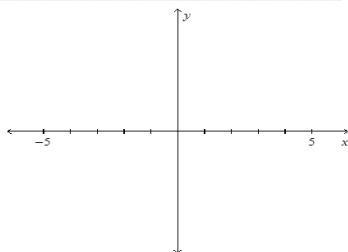
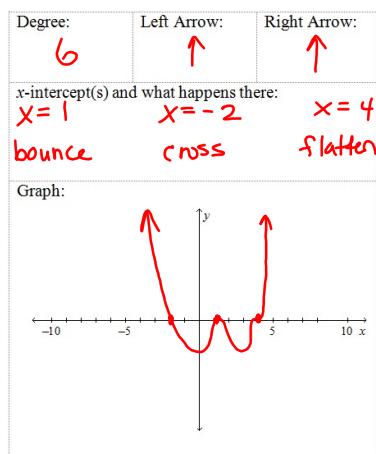
... when the factor is $(x \pm \#)^{\text{odd exponent}}$,
the graph flattens the x -axis.

5. Sketch rough graphs of the following polynomial functions without using a calculator.

a. $P(x) = -x(x + 1)^2(x - 3)$



b. $P(x) = (x - 1)^2(x + 2)(x - 4)^3$

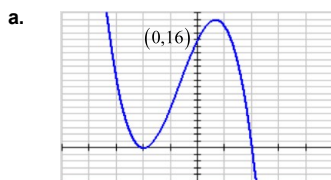


- c. What is the difference between the **graphs** of the functions:

$$y = x^2(x - 3)(x + 1) \text{ and } y = 3x^2(x - 3)(x + 1) ?$$

3
↓
vertical stretch

6. Some polynomials have a stretch factor, just like the a in parabolas and other parent functions. Write an exact equation (including the stretch factor) for each of the graphs.



$x = -2$ bounce

$x = 2$ cross

$$y = a(x+2)^2(x-2)$$

$$16 = a(0+2)^2(0-2)$$

$$16 = a(4)(-2)$$

$$16 = -8a$$

$$a = -2$$

$$y = -2(x+2)^2(x-2)$$

- b. Write a polynomial equation for a function with a graph that bounces off the x-axis at $(-1,0)$, crosses the x-axis at $(4,0)$, and goes through the point $(-2,-18)$.

↓
find a

$$y = a(x+1)^2(x-4)$$

$$-18 = a(-2+1)^2(-2-4)$$

$$-18 = a(1)(-6)$$

$$-18 = -6a$$

$$a = 3$$

$$y = 3(x+1)^2(x-4)$$